

The Transformation Problem of Values into Prices: From the Law of Value to Economic Planning

Tiago Camarinha Lopes
Universidade Federal de Goiás

ABSTRACT: The controversy about the transformation problem of values into production prices can be understood as a long debate in the history of economic thought that departs from the law of value and arrives at socialist planning. In order to defend this view, the paper distinguishes and describes three phases of the debate: the Engels Challenge from 1885 to 1906, the Traditional Transformation Problem from 1906 to 1971 and the Critique of Redundancy from 1971 onwards. It shows how the topic formally began and explains how each phase developed into the next. A table summarizing the main aspects of each phase is presented for illustration. Opposing the common idea that the controversy on the transformation problem does not advance, this paper argues that the debate originated from the challenge of the conciliation of the law of value with an equal average rate of profit, shows evolution in the long run because it forces Marxist and non-Marxist economic schools to confront the quality side of value in theory and to develop abstract models of planned economy in practice.

KEYWORDS: value and price, transformation problem, labour theory of value, plan and market

Introduction¹

In the beginning of the 21st century there is still no general accepted solution for the transformation problem of values into production prices. Having started officially at the end of the 19th century, it is one of the most controversial debates in theoretical Political Economy. It is common to hear that the discussion suffers from circularity, since it seems to deliver no satisfactory results. In opposition to this view, this paper presents the debate in its historical context and argues that the discussion achieves scientific progress when one considers its development in the long run.

The main aim of the article is to argue that the transformation problem obliges economists to deal with the relationship of two entities: the law of value and economic planning. In order to achieve this, the paper distinguishes the traditional transformation

problem from the broader problematic in which it is inserted. So, the advance of the discussion is shown by the confirmation that the traditional approach to the problem is only a specific episode within the broader controversy. Three concepts will be analyzed: the definition of the transformation problem, the character of the critique on Marxist economics and the subject under study. If these concepts have suffered positive changes in the last 125 years, the gradual advance of the debate will become evident and it will be shown that economists dealing with the topic must relate the law of value with planning.

For an adequate analysis of the historical phases of the debate, it is necessary to depart from an abstract presentation. The transformation problem describes the relation between value and price. While the word value is a theoretical category in economic science, which lies at a relatively high level of abstraction, the

¹ This article originally appeared as "As fases históricas do debate sobre a transformação dos valores em preços de produção" in *Revista de Economia Política*, 2012. See Lopes 2012. Translation by the author.

expression price is closer to reality. Both terms refer to the exchange relation of commodities – value, *theoretically* and price, *empirically*.² From this perspective, the development from the abstract to the concrete along the three volumes of *Das Kapital* is a parallel to the transformation of values into prices, and in the ideal tendency, into production prices. The starting point of the discussion is the following question: How does an equal average profit rate come about based on the law of value? There are two main methods of answering this challenge posed by Engels (1963) in the preface to *Capital* book 2.

The first method is based on the thesis of the contradiction between *Capital* book 1 and *Capital* book 3, represented by Eugen von Böhm-Bawerk (2007). It contains a negative critique of the economics of Karl Marx because it completely rejects *Capital* as a scientific theory capable of explaining the capitalist mode of production. The solution to the problem, according to this line, is impossible.

The second method of treating the subject goes back to the works of Ladislaus von Bortkiewicz (2007). In opposition to the fundamental denial, this path tries to follow the structure of *Capital* through the formalization and specification of the presentation. Therefore, it allows a more detailed study of the work and represents the continuation of the discussion about the mystery of the equal average profit rate. During the 20th century, different publications on the matter followed this line.

In the flow of the revival of Marxist analysis in the late 60s, the resurrection of the theme was due to the work of Piero Sraffa. Despite the fact that *Production of Commodities by Means of Commodities* was written to criticize the marginalist approach of the neoclassical school, it was rapidly discovered that the rehabilitation of Classical Political Economy (Meek 1961) compelled modern economic science to confront Marx. But the use of Sraffa's model to solve the transformation problem had a contradictory result: the question itself lost its meaning. At that moment, the thesis of the redundancy of the labour theory of value, already anticipated by Engels in the preface of *Capital* 3, was confirmed and broadly accepted. Therefore there were efforts to

end the debate. Surprisingly, the matter continued in a new form: now, the search for the economic meaning of the transformation became the central question.

Given that the discussion is long and does not have a clear solution, it is necessary to organize it historically. This necessity is pointed out by Schefold (1979) when he stresses the general incomprehension about the relation between Marx and Sraffa and the difficulty to explain it. By analyzing the publications on the problem it is possible to recognize three historical phases which help to conceive a totalizing presentation for the debate on the transformation of values into production prices. On the basis of this organization it is possible to understand that the conversion of values into prices resembles the concrete efforts of controlling the resources of the economy. In that sense, the economic planning in general and the socialist planning in particular are necessary results of the advancements of the debate on the transformation problem.

The article describes the historical formation of the debate by presenting three development sections, one for each historical phase. A summary table with the main characteristics of each period is shown and a short conclusion recalling the three concepts under investigation closes the paper.

Engels Challenge

At first sight the beginning of the history of the debate about the conversion from values into production prices is the correction made by von Bortkiewicz of an eventual mistake of Marx (2004). Actually, the debate starts earlier, with the publication of *Capital* 2.

In the preface, Engels poses the contradiction of Classical Political Economy as a challenge to theoreticians. It consists of the demonstration that an equal average rate of profit can and must come about, not only without a violation of the law of value, but on the very basis of it (Engels 1963). Engels believed that Marx had already solved the problem. However Marx (2004) was not able to clarify the issue entirely as the forthcoming alternative solutions showed.

One of these solutions was proposed by Lexis (1895). It is interesting to note that his answer to the challenge anticipates the critique of redundancy of the labour theory of value by more than half a century. According to him, the solution to the enigma of the

² This conception seems to be the starting point for Marx according to Rojas (1989).

equalized profit rates could only be achieved if, for the individual commodity, the measure of value as labour time was abandoned. He argued that, from this isolated perspective, there seems to be no comprehensible connection between price and the labour time required to produce the commodity. Yet Lexis (1895) is very careful with his conclusions: he does not state that prices cannot be measured and explained by labour time. He admits that real prices can be thought of as the point of arrival of the unities of labour time, in a process of transition. This observation calls attention because it points to the implicit function of the labour theory of value in the quantitative determination of exchange relations in the market. He concedes that prices can be understood as a form of appearance of labour-values, *but he also stresses that no one is obliged to use this point of view*. For that reason, Engels (2004) describes Lexis as a Marxist disguised as a vulgar economist.

Engels asserts that the theory of Karl Marx and vulgar economics explain exactly the same real phenomena, but with different arguments. As a reaction to the thesis of redundancy unconsciously defended by Lexis, Engels simply compares the concurrent propositions to explain the origin of profit. Since one of the aims of *Capital* 3 was to make Marx's theory comparable to other economic explanations, this aim seems to have been achieved. Therefore, we can suspect that the argument that the analysis of value made in *Capital* book 1 is unnecessary is the expected conclusion made by vulgar economists when studying Marx's *magnum opus*.

Apart from Lexis, other participants tried to contribute to the debate. The way of facing the problem around the end of the 19th century was to describe the operation of the law of value together with the process of competition. The debate centered on the comprehension of the fact that the law dominates the price movements and each participant tried to link the concept of the labour theory of value with the average rate of profit. The labour theory of value, being widely accepted, did not need to be firstly justified. The discussion aimed at understanding the theory of value of Marx from the standpoint of David Ricardo.

The continuity between Classical Political Economy and its Critique are involved in the transformation problem since its original formulation. From then on, the difficulties grew in complexity towards various

directions. Engels (2003) wanted to demonstrate the existence of the law of value with his controversial historical interpretation of *Capital* in order to clarify the reigning confusion. Although for him the solution of Marx was the correct one, there was a great pressure for an explanation of the issue after the publication of book 3. The real economic process behind the equalization of the profit rate based on the labour theory of value had to be better explained. Therefore, *Capital* book 3 did not clearly solve the paradox.

Those who were against the solution of Marx became both disappointed and satisfied with this situation according to Engels (2003). For this reason, the difference to David Ricardo regarding the determination of the quantity of value needed to be explicitly shown. This specific difference would be the technical improvement of the Ricardian labour theory of value, not being part of the Critique of Political Economy in its qualitative sense. This shows that there is an important continuity between classical economics and Marx's contribution, as many have pointed out after the publication of Sraffa (1960).

According to Engels (2003), when economists speak of value, they mean value really established in exchange. Marx, differently, when speaking of value, frequently means individual value, a quantity which is not defined only in the circulation process, but already in the production sphere. In the context of the transformation, the relation between the spheres of production and circulation originates lots of incomprehension. As Antonio de Paula (2000) rightly asserts, only the dialectical thinking about both sides can clarify how the size of the real exchange value is determined. The connection of individual value and social value occurs exactly in parallel to the linking of production with circulation. Then, the Marxist theory of value has its roots in the formulations of Ricardo and is, in the strictly quantitative sense, a technical improvement. The object under study is quantity of value. As in Classical Political Economy value can only be an expression of labour time the question on the equal average profit rate based on the law of value is solidly put forward by Marx. The quantitative side of the transformation is seen as solved after Sraffa (1960). Therefore, the third phase of the debate has a very different form than the first one: it is asked about the content of this quantity.

It is known that the quality question was exposed and solved by Marx in the beginning of *Das Kapital*. This question represents the singularity of the Critique of Political Economy: why is the measure of value an expression of labour time? One may note that it is very similar to the problem emerging from the quantitative solution after the Sraffa-shock: labour cannot have a differentiated function when we treat the economy only in its use-value structure. This is exactly the reason why the debate on the transformation problem also moves towards the pivot on which a clear comprehension of political economy turns (Marx, 1985).³

In sum, the opposing position to the Marxist school defended the position that there was an insoluble contradiction between the value system characteristic of *Capital* book 1 and the system of prices presented in *Capital* book 3. The critique of redundancy (unable to establish itself at that moment, but already recognized by Engels (2004) as a potential headache) was postponed. At this stage no conclusion was formulated. The debate went on and changed after the intervention of von Bortkiewicz (2007). A second phase emerged, the transformation problem as it became internationally known.

The Traditional Transformation Problem

Marx departed from Ricardo in order to overcome the contradiction of Political Economy: why does the quantity of labour time to produce a certain item not necessarily coincide with the price by which this very item is bought and sold? Answer: the law of value dominates the movement of real prices. This means that although prices are not directly proportional to individual values there is a mechanism that explains that deviation. However, when trying to present a mathematical formalization for this description, Marx could not arrive at a complete formal procedure. As Heinrich (1999) argues, since then, the problem is no longer seen as a failure of Classical Political Economy, but as an error of Marx himself.

Marx's quantitative solution in *Capital* book 3 counts the cost prices in terms of value. This condition guarantees that capitalists can buy the commodities composing the constant and variable capital to their individual values. Even though it is a possible case (if,

for example, the organic composition of the respective sectors producing the inputs at question were exactly equal to the average organic composition of the economy), in general, the prices of the commodities composing the constant and variable capital will be different from their individual values. Marx (2004) recognizes the limitations of his exposition and warns the reader that an ideal formalization should take that into account.⁴

The quantitative method had to be modified so that all possible cases of organic composition could be included in the model. Ladislaus von Bortkiewicz (2007), following Tugan-Baranowsky and probably influenced by Wolfgang Mühlpfordt, presents a solution of enormous repercussion in this direction.⁵

The research over the publications on the theme reveals that von Bortkiewicz's procedure establishes a starting point for the search for an algorithm of converting values into production prices. Accordingly, the quantitative side of the matter is the main area of study of the rest of the article. In this phase, the debate is clearly less ideological and polarized. However, it is not possible to recognize within it a clear path of progress or continuity. This is the reason why the transformation problem seemed to be circular. Indeed, only one aspect united the participants: the problem existed.⁶

It did not matter if the problem had been artificially created by the modification of Bortkiewicz, if it emerged from the limitations of Marx's solution, or if it was a product of the Ricardian theory of value. The fact is that the question was disseminated in this specific context, which contributed to spread the idea that the transformation problem was only about the formal quantitative conversion of one system into the other.

4 Fine and Saad-Filho (2010) also correctly advert that the limitation of Marx's procedure is real and that he was aware of it.

5 The reference to Bortkiewicz (2007) as the initiator of the traditional transformation problem must be maintained because almost all authors of that time entered in the discussion because of his contribution. According to Quaas (1992), Mühlpfordt (1893) had found a formal solution to the mystery of the average equal profit rate even before the publication of *Capital* 3. Apart from Tugan-Baranowsky and Mühlpfordt, V. K. Dmitriev and the mathematician Georg von Charassof participated in the first formalizations of the labour theory of value. On the origin of the traditional transformation problem, see Howard and King (1992), Quaas (1992) and Schefold (2004). Sraffa frontally criticized the method employed by Bortkiewicz in notes not intended for publication. For further information on this, see Gehrke and Kurz (2006).

6 Except for those who always considered it to be a spurious problem, like Samuelson.

3 This movement is best expressed in the qualitative development approach, which is presented in the section of the third phase of the debate.

The phase of the traditional transformation problem is specific in one sense: it has a largely accepted conclusion. This was possible because of the arguments developed on the basis of Sraffa (1960). The result can be summarized like this: the system of values is different from the system of production prices. This difference is due to the criterion of surplus-value distribution. In the system of values, the criterion is the size of the variable capital (the surplus is distributed in proportion to the quantity of living labour in each sector), while in the system of production prices the surplus the sum of the variable with the constant capital acts as the parameter of surplus distribution.

The so-called transformation of values into production prices is the passage from one system to the other. It is formally described through the multiplication of the system of values with a specific matrix, which reorganizes the distribution of surplus value so that the production prices emerge together with the profit proportional to the size of each capital, regardless of how it is constituted of variable and constant capital. By the way, the inverse of such a matrix allows the inverse transformation. The pure mathematical conversion of production prices into values is therefore also possible. This means that there is a mutual quantitative determination between the systems. Now, this result is no longer a source of controversies. However, the debates on the significance or interpretation of such a transformation did not end.⁷

After Samuelson (1971) presented his eraser-algorithm, a new controversy arose. Since then, the conflict between Marxists and Sraffians grew. Curiously, Samuelson started being attacked by the Marxist side, faithful to the labour theory of value, as well as by anti-Marxists.

Lerner (1972), for example, criticized Samuelson because he was making illegal concessions to the labour theory of value. The theory would be reusable if it was described merely as redundant. Lerner's panic due to Samuelson's concessions reveal how the critique from Böhm-Bawerk failed and the labour theory of value became acceptable many years after the complete dissolution of Classical Economics. In the past, the Marxist theory of value had been rejected because it was sup-

posedly wrong. But now, it became logically acceptable. The change from the accusation of contradiction to that of redundancy reveals that Marxist economics gained strength along the development of the debate.

Another reaction to Samuelson (1971) was Southworth (1972). He searches the motives for the increasing interest in Marx among economists and the reasons for their difficulties in comprehending his theory. Part of the confusion would be due to Marx's own texts, but another part would be the result of different methodologies of investigation. Southworth (1972) asks then if such a maneuver made by Samuelson is not a conscious strategy. This would be similar to that idea raised by Engels, that Lexis was a Marxist disguised as vulgar economist. Southworth (1972) argues that, in order to oppose the increasing credibility of Marxist economic theory, some papers have the political aim of offsetting the return of Marxist analysis within economic science. Samuelson's *Voilà!* would be an example of such actions, as well as his qualification of Marx, the economist, as a minor Post-Ricardian.

Indeed, Samuelson (1971) showed clearly that values could be converted into production prices with the help of the already presented method. This enabled the construction of arguments departing from both extremes: from values and from prices. On one side, the analysis of values was said to be an unnecessary detour. On the other side, it was argued that prices were entities without meaning if not linked to values. The polarization of the debate became very clear after the dissolution of the traditional transformation problem.

So the algorithm had been found, and the problem had been solved (or extinguished). Still, papers on the matter continued to appear. Why? Because as time passed and the debate grew on formality, the origin of the challenge was lost. By the time a solution was found, the problem was no longer comprehended in its totality.

Naturally, the debate should turn back to the origins of the problem. And this was exactly what happened. Baumol's (1974) article represents a new form of facing the issue. He argues that the authors since Bortkiewicz gave too much attention to a theme which was marginal for Marx. What Marx really meant was that the distribution of surplus in different forms of income of capital through competition could be

⁷ For a formal summary of the result allowed by Sraffa (1960) see Pasetti (1979).

illustrated by a mathematical model. Since 1907, the search for such a model demanded all efforts from participants. But what really mattered was to know how the surplus value was distributed in reality. Besides, Baumol (1974) emphasizes that Marx knew that production prices could be quantitatively determined without any mention to values. Therefore, the crucial movement would be the transformation of the surplus value into the different categories of remuneration of capital property, like profit and interest. This conception of the problem differs substantially from the traditional approach.

Up from the point when a mathematical operation of conversion was broadly accepted, new forms of treating the issue began to appear. Since then, many studies look for a meaning of the transformation. In 1977, Ian Steedman published *Marx after Sraffa*. He urges economists to show the necessity and usefulness of the labour theory of value for a materialist analysis of the economy. This officially initiated a new phase for Marxists economists.

This means that the traditional transformation problem was only a specific episode inside a broader question involving the continuity and rupture of Political Economy. The practical result of the traditional problem is the definitive refutation of the critique of contradiction between *Capital* book 1 and *Capital* book 3. It is therefore secure to state that the critique of Böhm-Bawerk (2007) was completely debunked. The critique of contradiction was thus substituted by the critique of redundancy in this particular context.

The Critique of Redundancy

Different alternatives came as an answer to the Sraffa shock in order to address the redundancy of the analysis of values. There is disagreement on how to confront this situation. For that reason, the third phase of the debate is fundamentally a dispute between theoreticians of the labour theory of value. This is the reason why the communication with non-Marxists schools became limited in recent years. Among the alternatives of the new scenario are the new solution, the temporal

single system, the qualitative development and the probabilistic approximation.⁸

The New Solution

The New Solution or New Interpretation was the first alternative approach with strong influence. Duménil (1983-1984), Foley (1982), and Lipietz (1982) were those who put this interpretation forward.

Duménil (1983-1984) argues that the relevance of the labour theory of value is at stake. According to him, the concept of value is a theoretical necessity, because of the aggregation of different use values. So the labour theory of value would have a specific clarifying function: to explain the social division of labour in a historical context, while the theory of price could only describe this division in societies that produce commodities. Furthermore, for Duménil, the New Solution has to be defended with citations from Marx, so it cannot be freely developed without base from the original text. On the other hand, he recognizes that the interpretation of redundancy can also be demonstrated from Marx's writings. This would indicate ambivalences in the text that should be carefully analyzed.

Although the New Solution seems to be organized, it is not possible to recognize a counter critique to the argument of redundancy at this stage. For some time it was the most popular alternative among labour value theoreticians,⁹ but it is being criticized in the last years. Some simply reject it,¹⁰ while others show themselves sympathetic to its objective of defying the dominance of the redundancy solution to the transformation problem.¹¹ So we may say that the New Solution was a pioneer in the political unifying sense, but its fragile theoretical proposition could not be maintained.

The Temporal Single System Interpretation

Another reaction stream is called Temporal Single System Interpretation (TSSI).¹² According to Howard

⁸ This is a representative selection of reactions which aims to illustrate the diversity of interpretations developed from the Sraffa-shock.

⁹ Glick and Ehbar (1987) and Campbell (1997) are examples of acceptance of the New Solution.

¹⁰ For example Sinha (1997).

¹¹ For example Moseley (2000), Fine, Lapavistas and Saad-Filho (2004) and Gontijo (2006).

¹² For presentations of the TSSI by its proposers, see Kliman and McGlone (1999), Freeman, Kliman and Wells (2004) and Kliman (2007). On my part, I do not think the TSSI is a wrong interpretation, though I

and King (1992), it started being developed in the 1980s, but it gained popularity only later. It recognizes itself as an alternative to all methods that link values to prices in a simultaneous form. So for proponents of the TSSI, all discussion based on models of simultaneous transformation (including Dmitriev, Bortkiewicz, Sweezy, Seton, Okishio, Morishima, Shaikh, Steedman, and Laibman) cannot explain the meaning of the conversion of values in prices.

In the simultaneous interpretation (the traditional), the transformation does not occur in steps: the value system and the price system are determined at the same time. There are consequently two sets of exchange relations. One known conclusion from this interpretation is the possibility to choose for one of the systems, so that there is no theoretical connection between them. According to Freeman, Kliman and Wells (2004), this led to the separation of the systems and turned the issue into a spurious problem. The redundancy of the value system and the abandonment of Marxian economics were practical results from this perspective.

In the TSSI, on the contrary, the transformation takes place in a chronological order. Firstly, there is the value system, which is determined by the technical conditions of the economy and secondly, the price system is derived from this origin. Then, a temporal connection between both systems emerges, similar to Marx's formulation. Furthermore, the assumption of the equal average profit rate is studied and it is said that it can only be thought of as a tendency.¹³ So the TSSI has the objective to put Marx's own solution in debate in order to clarify the meaning of the transformation. The TSSI actually refutes the correction of von Bortkiewicz with the statement that since then, this erroneous interpretation has dominated the debate.

Authors who criticize the TSSI see in it a further frustrated attempt to save Marx. Indeed, it is not clear why the TSSI denies the result of the redundancy of the value analysis. For this reason, Gary Mongiovi (2002)

believe it employs a wrong strategy for solving the controversy. The task is to recover the Cambridge-Cambridge Critique and to unite the Marxian and the Sraffian Schools against mainstream neoclassics. This is my guess on how to reverse the disintegration of the Marxian school as described by Kliman (2010).

¹³ This conclusion is very close to the main argument of Farjoun and Machover (1983). Both methods are temporal, opposing the traditional simultaneous method. On the relationship of the TSSI and Farjoun and Machover, see Wells (2007).

affirms that what the TSSI really lacks is a lucid explanation of why we, after all, need the labour theory of value after Sraffa.¹⁴

More precisely, the TSSI seems to confuse the critique of contradiction with the critique of redundancy. Because of this, it seems to fall again into the trap of trying to justify the labour theory of value. In one word: all efforts of the TSSI are directed to sustain that Marx's theory of value is consistent. But this is not in dispute anymore! Opponents of Marxian economics argue now, differently from Böhm-Bawerk, that the theory is redundant, precisely because it was shown that it is internally consistent.

The Qualitative Development

The approximation qualitative development was strongly influenced by the pioneer works of Isaak Rubin (1978). The authors of this tradition emphasize the qualitative analysis of value in order to solve the dilemma. Here, the concept of abstract labour is studied with great caution. So the qualitative side of the value analysis becomes the focus of attention. The concept of Critique of Political Economy becomes central and the difference between Marx and the classical economists are made clear. As a result, the determinations of the quantity of value are neglected. The link between value and price would rather be a development of categories, a connection between expressions on different levels of abstraction, and not a logical-mathematical procedure. The quantitative conversion of values into production prices would be an incorrect method of dealing with the problem, according to this approach.

Why did Marx point out to the mathematical direction then? Heinrich (1999) argues that, although Marx initiated a new area of investigation, some elements of Classical Political Economy unwillingly continue to exist in his own writing. This means that Marx still uses classic concepts which would not be compatible with the new system of Political Economy.

This argument is similar to that of Paul Mattick (1974), who believes that the empirical research on the exchange relations has low practical relevance. Connecting prices to individual values is not so important in his view. It is more relevant to observe the total

¹⁴ For recent critiques of the TSSI see also Mohun (2003) and Park (2009) and the replies by Freeman and Kliman (2006) and Kliman (2009).

fall or rise of the sum of production prices, because that would bring knowledge about the development of the productive forces of society.

Another approach from the perspective of quality is developed by Fine and Saad-Filho (2010). They deal with the implications of the quantitative solution by arguing that more significant than the algebraic solution of the transformation problem is the observation that Marx's theory of value is not fully grasped by some specific algebraic solution separated from the totality of Marx's contribution to science (Fine and Saad-Filho (2010, 112)). The crucial point, they argue, is that value exists as a consequence of the social relations. Thus, instead of having to choose between either the value or the price system, Fine and Saad-Filho (2010) claim that the relationship between them must be theoretically recognized and analytically explored.¹⁵

Many other authors can be classified in the qualitative development category, such as Coutinho (1974) and Belluzzo (1998), who pioneered the debate in Brazil for example. However, it is improbable that this kind of interpretation will be able to end the discussion if the quantitative developments made along the years do not find their rationale. It is important to remember here that the quantitative aspect of the theory of value has strong support from Marx, who was particularly interested in explaining the quantitative relation between value and price. More importantly, dialectics as a method of investigation disallows the focus only on the quantitative or the qualitative aspect of value theory.

The Probabilistic Approach

From all these alternatives, the probabilistic method deserves attention because it answered to Steedman (1977) most directly. Farjoun and Machover (1983) developed a statistical approach to verify postulates

of Political Economy with the purpose to solve the dilemma on the transformation problem. For them, a fundamental assumption of the problem is at the same time a misleading one: the assumption of the equal average profit rate.

They criticize the determinism that emerges from this and present a non-determinist model, where the profit rate can only be given probabilistically. The traditional methods consider that the profits of all sectors are equal because profit rate equalization through competition is a true experience which appears as a reality for the theoretician. Actually, the profit rates are all different, as the empirical results show and the market analysts know.

Despite Marx being aware of this, he treats the profit rates as if they had equalized. The problem of observing the equal average profit rate as something real lies in the inversion of the transformation. Values can then be derived from prices. Marx warned that the assumption of the equal average profit rate could lead to this erroneous conclusion. For that reason he stressed that, even when the assumption is made, there is only one possible direction of transforming, which is from values towards prices. The probabilistic approach can be summarized as follows.

The exchange relations on the market are adjusted to the production prices through the process of free market. The price of a commodity can be freely negotiated between buyer and seller, but the variability of prices are limited by unknown parameters. Prices of production are not the object of study in this approach, in opposition to Sraffa: there is only the value system (individual values in production) and the market prices (observable exchange relations formed on the market). Marx writes that the concrete exchange relations are governed by economic laws, specially the law of value. This means that, even though the market prices are open (they appear to be negotiated without any bounds), the law of value dominates and regulates them. The connection between value and price can be modelled with probabilistic calculations where there is no absolute tendency for equilibrium, as shown in Figure 1.

Considering this summary of Farjoun and Machover's approach, it is possible to comprehend

¹⁵ So they do not reject the quantitative analysis, but call attention to the fact that it must be integrated to some meaning regarding both value and price system. This relationship should clearly determine the differences between the value and organic composition of capital, as Fine (1983) has initially outlined. On the value, organic and technical composition of capital, see Fine (1990). On the consequences of this categorical development of the capital composition for the transformation problem, see Saad-Filho (1997). On the attempt of Ben Fine and Alfredo Saad-Filho to present the dialectics of the quantity and quality in value theory, see Fine and Saad-Filho (2009).

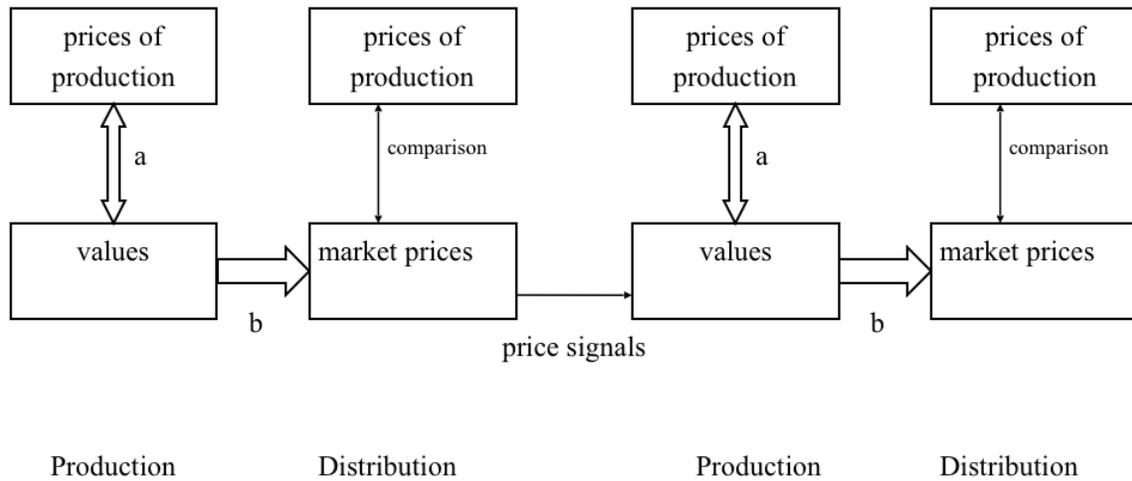


Figure 1: Values, prices of production and market prices

the meaning of the traditional algorithm. The starting point of this presentation is the value system (box labelled values).¹⁶ These are the individual values of commodities. In other words, they are the individual labour time used to produce each individual product. This system represents the technical reality and it is inaccessible information in societies which socialize individual labour through the market. It is determined by the own technical structure given by the productive forces of society. This combination of values does not allow in reality an equal profit rate for capitals with the same size but with different organic compositions.

The traditional solution is represented by the double arrow “a”: it is an algorithm that enables a new distribution of the surplus value according to the equal average profit rate. Accordingly, the profit rate in Sraffa’s model and the production prices are determined by the use-value structure of production. This solution must be carefully interpreted: as there is reciprocity between both systems, it is said that values are not the starting point towards production prices.¹⁷ But, as circulation is not yet in the picture, it is clear that on this stage, one is not dealing with concrete exchange relations.

Alternative “b” represents the probabilistic method.

The adjustment of values to production prices takes place only through a trial-and-error process typical of the market. Therefore, production prices and the average profit rate can only be an abstract creation, an objective around which the production finds orientation. The transformation “b” goes from the value system towards the market prices system without a deterministic calculation. Moreover, market prices are the only way to get access to the technical reality of the private productive unity. In other words: technical conditions of production, under capitalist relations, show themselves only through the market process. Production data (input quantity, including labour) are private property in the capitalist mode of production and are socialized only afterwards on the market. Although these data exist, we do not see them prior to circulation. These data would be visible beforehand only through nationalization and/or planning.¹⁸ Therefore, market prices act as empirical information about the technical conditions of production in capitalism.

In this sense, value and price correspond to those two levels of observing exchange relations. Accordingly, values are not only theoretically but also temporally

¹⁶ Each box represents a system. So values represents the value system, prices of production represents the production prices system and market prices are the empirical exchange relations which really took place.

¹⁷ This is the starting point for the formalization of the critique of redundancy developed by Samuelson (1971).

¹⁸ This is the reason why economic schools in general are led to build planned economy models when dealing with the transformation problem. For a Marxist position see, for example, Cockshot and Cotrell (1989), who currently develop Farjoun and Machover’s approach further and link the results with the economic calculation debate. For a presentation of methods of analyzing the variable profit rate in accordance to this approach, see Wells (2007).

	Phase 1: Engels' challenge (1885-1906)	Phase 2: The traditional transformation problem (1906-1971)	Phase 3: The critique of redundancy (1971 – today)
Main theme	Law of value and the equal average profit rate	Mathematical formulae	Labour theory of value
Critique	Contradiction between value system and price system	Marx's quantitative solution	Redundancy of the value system
Function of transformation	Equalization of profit rates based on the law of value	Conversion of values into production prices	Distribution of surplus value / market simulation
Starting point	Engel's challenge	Marx's formalization wrong/incomplete	Relation between transformation and planning
Sraffa's effect	Difference between Marx and Classical Political Economy raises	end	Confirmation of the quantitative redundancy of the labour theory of value
Conclusion	–	Contradiction between value system and production price system explained (quantitative problem solved)	–
Next researches	Relation between Marx and Sraffa/Sraffian school	–	Origin/utility of the labour theory of value
Formal question	How does an equal average profit rate is formed based on the law of value?	How is it possible to model the conversion of values into production prices?	–

Figure 2: A summary table showing the historical phases of the debate on the transformation of values into prices. The three historical phases of the discussion are displayed side by side, so that a simple comparison can be made. The organization of the table does not imply that the themes are strictly separated. It means that each period had specific characteristics.

prior to prices, as Marx argued. Bettelheim (1969) remembers that the differences between the price theories and the Marxian framework are not visible on the practical level of economic calculation. This supports the idea that economic schools away from Marx, when dealing with the transformation problem, may indirectly produce technical material for the praxis of planning.

The relation between “a” and “b” becomes clear with the passage from the production price system to the sphere of circulation. Alternative “a” would be the theoretical transformation, representing the algorithm of conversion which expresses the result of competition.

This was the focus of the traditional transformation problem. Alternative “b” on the other hand, would be the transformation on a more concrete level.

Prices of production represent the objective of the movement and the assumption of the equal average profit rate: after the market prices are formed on the circulation sphere, this system is compared with the system of production prices. If they are different, an equal average profit rate was not achieved. This is the reason for a new allocation of factors in the next period. The system of values will transform itself in the new system (values). The importance of this constant comparison and reallocation is expressed in the relevance

of the price signs or in the relation between supply and demand.

This should partially indicate how the economic calculation debate and the operation of the law of value in the planned economy can be put in connection to the traditional transformation problem.

The Progress of the Debate

According to the present study, the traditional transformation problem lies within a greater problematic involving the questions of continuity and rupture in Classical Political Economy. With respect to those three concepts under analysis, three results can be here summarized.

First, it is necessary to broaden the definition of transformation of values into production prices, since the debate deals with a vast field of analysis in theoretical political economy. Therefore, a strict definition of the theme makes only sense when one wants to observe a specific phase of the debate. The classical conception of the transformation is related exclusively to the phase of the traditional transformation problem, which encompasses only the problem of quantitative conversion.

Second, as time passes the emphasis on certain aspects of the issue is different. Mainly, the form of the critique on Marxist economics suffered a significant change. From the perspective of theory the progress is expressed in the failure of the Böhm-Bawerk-type of attack. Here we have an important result. Because the argument opposing Marxist economic theory has moved from an accusation of contradiction to one of redundancy, it becomes very visible that any rejection of the economic theory presented in *Capital* is a matter of political choice, not of technique. The debate was strongly polarized after Samuelson (1971) precisely because of this.

Third, the analysis of the value form became the central subject under study after the implications of the model of Sraffa (1960). At this moment, the economic meaning of the transformation started to gain attention. Now, old quantitative solutions are opening space for the formulation of a new questioning of qualitative nature. As the content expressed in the relations of quantitative exchange became empty after Sraffa, the discussion was directed to the qualitative analysis

of value. Moreover, the relationship between Marx and Sraffa became one of the main questions which does not have a full accepted answer in the literature (Bellofiore 2008).

This last point demands perhaps a more detailed explanation, since it may be the most fragile of these three results. As Schefold (1974) had warned, the Critique of Political Economy encounters tough resistance every time the theoreticians are compelled to deal with the qualitative side of the theory of value. The traditional economic theory systematically hinders the study about the quality of value by concentrating all efforts on the quantitative side. Because of this, even though the contemporary phase of the debate on the transformation problem calls for the qualitative analysis of value theory, economists distant from the Marxist tradition will abandon the debate by (correctly) arguing that it is useless for the quantitative measure of wealth. This is the context in which the critique of redundancy of the labour theory of value should be understood. In the turn to the 21st century, there can be no discussion similar to phase 1 because today, the law of value in all of its complexity is not generally accepted as a subject of study in economics. In the end of the 19th century there were no doubts that values needed to be somehow linked to prices since the labour theory of value was broadly accepted before the dissolution of the Ricardian school. Moreover, if the challenge posed by Engels is accepted, one cannot affirm that the redundancy-argument is a solution to the problem, since it is not based on the law of value. In this sense, it would be merely an answer to the traditional quantitative problem.

On the other hand, even though non-marxist schools do not consider the labour theory of value, they develop abstract models of planned economy when they conciliate both the value and price systems. By doing that, they unconsciously contribute to a better understanding about the capitalist economy and to the development of the theory of mixed economies. One of these results was the development of approximations that, in combination to input-output models, may offer new tools for the practice of planning.

So, as the quantitative problem is solved, the problematic must necessarily be directed to the Critique of Political Economy as a way to understand the

concept of value for Marx. And since value for him is very closely related to abstract labour, the debate on transformation problem needs to comprehend how this category was historically formed. But this must not neglect the aspect of the quantitatively concrete determination of prices, since it has practical validation in the field of planning. Only then economists will fulfill their scientific objective of treating the theory of value dialectically. Socialist planning is therefore a necessary outcome from the full development of the original problem of explaining how the law of value must be related to the real quantitative relations of exchange between use values observed in the reality.

Finally, it is shown that, due to the changes in the definition of the problem, in the character of the critique and in the subject under study, there is scientific progress in the debate. In this context, it is possible to comprehend that, even on an unorganized framework, the debate on the transformation problem is moving to the analysis of the value form and to the theory of economic planning.

References

- Antônio de Paula, J. 2000. "A Dialética Valores e Preços." *Revista de Economia Política*, vol. 20, no. 4 (80):116-136.
- Baumol, W. J. 1974. "The Transformation of Values: What Marx Really Meant (An Interpretation)." *Journal of Economic Literature*. 12 (1):51-62.
- Bellofiore, R. 2008. "Sraffa after Marx: an Open Issue." In *Sraffa or an Alternative Economics*, edited by Guglielmo Chiodi and Leonardo Ditta, 68-92. New York: Palgrave Macmillan.
- Belluzzo, L. G. 1999). *Valor e Capitalismo: Um Ensaio Sobre a Economia Política*. Campinas, SP, Unicamp, IE.
- Bettelheim, C. 1969. *A Transição para a Economia Socialista*. Zahar: Rio de Janeiro.
- Böhm-Bawerk, Eugen von. 2007. *Karl Marx and the Close of His System*. Auburn, Alabama: Ludwig von Mises Institute. First published 1896.
- Bortkiewicz, Ladislaus von. 2007. "On the Correction of Marx's Fundamental Theoretical Construction in the Third Volume of Capital." In *Karl Marx and the Close of His System*, edited by Eugen von Böhm-Bawerk. Auburn, Alabama: Ludwig von Mises Institute. First published 1907.
- Campbell, A. 1997. "The Transformation Problem: A Simple Presentation of the New Solution." *Review of Radical Political Economics* 29 (3):59-69.
- Clark, D. L. 1984. "Planning and the Real Origins of Input-Output Analysis." *Journal of Contemporary Asia* 14 (4):408-429.
- Cockshott, P. and A. Cottrell. 1989. "Labor Value and Socialist Economic Calculation." *Economy and Society* 18 (1):71-99.
- Coutinho, L. 1974. "Notas sobre o Problema da Transformação: uma crítica a Samuelson." *Estudos Cebrap* 9.
- Duménil, G. 1983-1984. "Beyond the Transformation Riddle: a Labor Theory of Value." *Science and Society* XLVII (4):427-450.
- Engels, Friedrich. 1963. "Vorwort." *Karl Marx - Friedrich Engels - Werke, Band 24, "Das Kapital", Bd. II*. [MEW 24] Dietz Verlag, Berlin/DDR, 1963. First published 1885.
- Engels, Friedrich. 2004. Vorwort. Das Kapital. Kritik der Politischen Ökonomie. Dritter Band. Hamburg 1894. In: *Karl Marx, Friedrich Engels, Gesamtausgabe. Zweite Abteilung: "Das Kapital" und Vorarbeiten, Bd. 15*. [MEGA, II, 15, Text], Berlin: Akademie Verlag, 2004, 5-23. ([1894])
- Engels, Friedrich. 2003 [1895-1896]. Wertgesetz und Profitrate. Erster Nachtrag zu Buch 3 des „Kapitals“. In: Marx, K.; Engels, F. (1871-1895). *Manuskripte und Redaktionelle Texte zum dritten Buch des „Kapitals“*, [MEGA, II, 14, Text, 2003], Akademie Verlag 2003.

- Farjoun, E. and M. Machover. 1983. *Laws of Chaos: A Probabilistic Approach To Political Economy*. London:Verso.
- Fine, Ben. 1983. "Note: A Dissenting Note on the Transformation Problem." *Economy and Society* 12 (4):520-525.
- Fine, Ben. 1990. "On the Composition of Capital: A Comment on Groll and Orzech." *History of Political Economy* 22 (1):149-155.
- Fine, Ben, C. Lapavistas and A. Saad-Filho. 2004. "Transforming the Transformation Problem: Why the New Interpretation is a Wrong Turning." *Review of Radical Political Economics* 36 (1):3-19.
- Fine, Ben and Alfredo Saad-Filho. 2009. "Twixt Ricardo and Rubin: Debating Kincaid Once More." *Historical Materialism*, vol. 17, 192-207.
- Fine, Ben and Alfredo Saad-Filho. 2010. *Marx's Capital* (Fifth Edition). London and New York: Pluto Press.
- Foley, D. K. 1982. "The Value of Money the Value of Labor Power and the Marxian Transformation Problem." *Review of Radical Political Economics* 14 (2):37-47.
- Freeman, Alan, Andrew Kliman and Julian Wells, eds. 2004. *The New Value Controversy and the Foundations of Economics*. Cheltenham, UK, Edward Elgar.
- Freeman, Alan and Andrew Kliman. 2006. "Replicating Marx: A Reply to Mohun." *Capital & Class* 88:117-126.
- Gehrke, C. and Kurz, H. D. 2006. "Sraffa on von Bortkiewicz: Reconstructing the Classical Theory of Value and Distribution." *History of Political Economy*, vol. 38, no. 1, 91-149.
- Glick, M. and Ehrbar, H. 1987. The Transformation Problem: An Obituary. *Australian Economic Papers*, 26 (December): 294-317.
- Gordon, S. 1968. "Why Does Marxian Exploitation Theory Require a Labor Theory of Value?" *Journal of Political Economy*, vol. 76, no. 1
- Gontijo, C. 2006. "A Nova Solução do Problema da Transformação de Valores em Preços e o Marxismo." *Economia*, Brasília (DF), 7, no. 2, 225-248.
- Harcourt, G. C. 1972. *Some Cambridge Controversies in the Theory of Capital*. Cambridge.
- Heinrich, M. 1999. *Die Wissenschaft vom Wert*. Münster.
- Howard, M. C. and King, J. E. 1992. *A History of Marxian Economics*. Macmillan.
- Kliman, Andrew. 2007. *Reclaiming Marx's Capital: A Refutation of The Myth of Inconsistency*. Lexington Books.
- Kliman, Andrew. 2009. "A Reply to Professor Parks Critique." *Marxism* 21 6(1):332-346.
- Kliman, Andrew. 2010. "The Disintegration of the Marxian School." *Capital & Class* 34(1):61-68.
- Kliman, Andrew and T. McGlone. 1999. "A Temporal Single-System Interpretation of Marx's Value Theory." *Review of Political Economy* 11 (1):33-59.
- Kurz, H. D. and N. Salvadori. 2003. *Classical Economics and Modern Theory: Studies in Long-Period Analysis*. New York: Routledge.
- Lerner, A. 1972. "A Note on Understanding the Marxian Notion of Exploitation." *Journal of Economic Literature* 10(1):50-51
- Lexis, W. 1895. "The Concluding Volume of Marx's Capital." *The Quarterly Journal of Economics* 10 (1):1-33.
- Lipietz, A. 1982. The So-Called Transformation Problem Revisited. *Journal of Economic Theory* 26 (1):59-88.
- Lopes, Tiago Camarinha. 2012. "As fases históricas do debate sobre a transformação dos valores em preços de produção." *Revista de Economia Política*. 2012 32(2):315-335. http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0101-31572012000200010
- Marx, Karl. 2004 [1872]. *Das Kapital. Kritik der politischen Ökonomie. Erstes Buch: Der Produktionsprozess des Kapitals*. Parkland Verlag.
- Marx, Karl. 2005 [1884/1885]. "Das Kapital. Kritik der Politischen Ökonomie. Zweites Buch. Redaktionsmanuskript von Friedrich Engels." *Karl Marx, Friedrich Engels, Gesamtausgabe. Zweite Abteilung: „Das Kapital“ und Vorarbeiten, Bd. 12*. [MEGA, II, 12, Text, 2005], Akademie Verlag.
- Marx, Karl. 2004 [1894]. "Das Kapital. Kritik der Politischen Ökonomie. Dritter Band. Hamburg 1894." *Karl Marx, Friedrich Engels, Gesamtausgabe. Zweite Abteilung: „Das Kapital“ und Vorarbeiten, Bd. 15*. [MEGA, II, 15, Text, 2004], Berlin: Akademie Verlag.
- Mattick, P. 1972. "Samuelson's Transformation of Marxism into Bourgeois Economics." *Science & Society* 36:258-273.
- Meek, Ronald L. 1956. "Some Notes on the Transformation Problem." *The Economic Journal* 66 (261):94-107.
- Meek, Ronald L. 1961. "Mr. Sraffa's Rehabilitation of Classical Economics." *Scottish Journal of Political Economy* 8 (2):119-136.
- Mohun, S. (Ed.). 1994. *Debates on Value Theory*. St. Martin's Press.
- Mohun, S. 2003. "On the TSSI and the Exploitation Theory of Profit." *Capital & Class* 81:86-102.
- Mongioli, G. 2002. Vulgar Economy in Marxian Garb: A Critique of Temporal Single System Marxism. *Review of Radical Political Economics* 34 (4):393-416.
- Morishima, Michio. 1973. *Marx's Economics: A Dual Theory of Value and Growth*. Cambridge University Press.

- Morishima, Michio and G. Catephores. 1975. "Is There 'An Historical Transformation Problem'?" *Economic Journal* 85(338):309-28.
- Moseley, F. 2000. "The New Solution to the Transformation Problem: A Sympathetic Critique." *Review of Radical Political Economics* 32 (2): 282-316.
- Mühlpfordt, W. 1893. *Preis und Einkommen in der privatkapitalistischen Wirtschaft*. Zugl. Diss. Königsberg 1893.
- Neto, João Machado Borges. 1997. *A Transformação dos Valores em Preços de Produção em um Sistema Único Temporal*. Dissertação de Mestrado, PUC/SP, 1997.
- Nguyen, D. 1982. "Notes on Professor Samuelson's Analysis of the Marxian Transformation Problem." *Southern Economic Journal* 49 (1): 1-10.
- Park, C-S. 2009. "The Temporal Single System: A Retrospective Critique." *Marxism* 21 6 (1): 318-331.
- Pasinetti, L. 1979. *Lectures on the Theory of Production*. Palgrave Macmillan.
- Quaas, F. 1992. *Das Transformationsproblem: Ein theoretischer Beitrag zur Analyse der Quellen und Resultate seiner Diskussion*. Marburg, Metropolis.
- Quaas, F. 2000. "Das Transformationsproblem von Werten in Produktionspreise: Ein Diskussionsbeitrag zur Geschichte und zum Stand der Debatte." *Jahrbücher für Nationalökonomie und Statistik* 220:108-121.
- Rakowitz, N. 2000. *Einfache Warenproduktion: Ideal und Ideologie*. Freiburg.
- Rojas, R. 1989. *Das unvollendete Projekt: zur Entstehungsgeschichte von Marx „Kapital“*. Berlin, Hamburg, Argument Verlag.
- Rubin, I. I. [1927] 1978. "Abstract Labor and Value in Marx's System." *Capital & Class* 5:107-39.
- Saad-Filho, A. 1997. "An Alternative Reading of the Transformation of Values into Prices of Production." *Capital & Class* 63: 115-136.
- Samuelson, P. A. 1957. "Wages and Interest: a Modern Dissection of Marxian Economic Models." *American Economic Review* 47:884-912.
- Samuelson, P. A. 1971. "Understanding the Marxian Notion of Exploitation: A Summary of the So-Called Transformation Problem Between Marxian Values and Competitive Prices." *Journal of Economic Literature* 9 (2): 399-431.
- Schefold, B. 1974. "Wert und Preis in der marxistischen und neoklassischen Akkumulationstheorie." *Mehrwert, Beiträge zur Kritik der Politischen Ökonomie*, vol. 2.
- Schefold, B. 1979. "Kapitaltheorie: Vom Transformationsproblem und der Kritik an der Neoklassik zur Rekonstruktion der Politischen Ökonomie." *Jahrbuch für Sozialwissenschaft* 30 (2): 177-188.
- Schefold, B. 2004. "Einführung. Der Dritte Band: Herkunft und Wirkung." In: *Karl Marx: Das Kapital. Kritik der politischen Ökonomie. Dritter Band*. Hamburg 1894. Berlin, Akademie Verlag, 2004. MEGA, II, 15: Karl Marx, Friedrich Engels, Gesamtausgabe. Zweite Abteilung: „Das Kapital“ und Vorarbeiten, Bd. 15, [MEGA, II, 15, Apparat, 2004], 871-910.
- Screpanti, E. and S. Zamagni. 1993. *An Outline of the History of Economic Thought*. Oxford.
- Seton, F. 1957. "The Transformation Problem." *The Review of Economic Studies* 24 (3): 149-160.
- Sinha, A. 1997. "The Transformation Problem: A Critique of the New Solution." *Review of Radical Political Economics* 29 (3): 51-58.
- Southworth, G. 1972. "Samuelson on Marx: A Note." *Review of Radical Political Economics* 4:103-111.
- Sraffa, Piero. 1960. *Production of Commodities by Means of Commodities: Prelude to a Critique of Economic Theory*. Cambridge: Cambridge University Press.
- Steedman, I. 1977. *Marx After Sraffa*. London.
- Wells, P. J. 2007. *The Rate of Profit as a Random Variable*. PhD thesis, School of Management, The Open University.